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Anti-periodic fractional boundary value problems

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ABSTRACT

We study a class of anti-periodic boundary value problems of fractional differential equations. Some existence and uniqueness results are obtained by applying some standard fixed point principles. Several examples are given to illustrate the results.

1. Introduction

In recent years, a variety of problems involving differential equations and inclusions of fractional order have been investigated by several researchers. Fractional differential equations appear naturally in a number of fields such as physics, chemistry, biology, economics, control theory, signal and image processing, biophysics, blood flow phenomena, aerodynamics, fitting of experimental data, etc. For theory and applications of fractional calculus, see [1–4]. Some recent works on fractional differential equations and inclusions can be found in a series of papers [5–13] and the references therein.

Anti-periodic boundary value problems occur in the mathematical modeling of a variety of physical processes and have recently received considerable attention. For examples and details of anti-periodic boundary conditions, see [14–19] and the references therein.

In this paper, we investigate the existence and uniqueness of solutions for an anti-periodic fractional boundary value problem given by

$$\begin{cases} {}^cD^q x(t) = f(t, x(t)), \quad t \in [0, T],\ T > 0,\ 1 < q \le 2, \\ x(0) = -x(T), \quad {}^cD^p x(0) = -{}^cD^p x(T),\ 0 < p < 1, \end{cases} \tag{1.1}$$

where ${}^cD^q$ denotes the Caputo fractional derivative of order q, and f is a given continuous function.

2. Preliminaries

Let us recall some basic definitions [1–3].

Definition 2.1. The Riemann–Liouville fractional integral of order q is defined as

$$I^q g(t) = \frac{1}{\Gamma(q)} \int_0^t \frac{g(s)}{(t-s)^{1-q}} ds, \quad q > 0,$$

provided the integral exists.

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