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Positive solutions of a nonlinear nth order boundary value problem with nonlocal conditions

Paul W. Eloe[a,*], Bashir Ahmad[b]

[a]*Department of Mathematics, University of Dayton, Dayton, OH 45469-2316, United States*
[b]*Department of Mathematics, King Abdul Aziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia*

Abstract

We discuss the existence of positive solutions of a nonlinear nth order boundary value problem

$$u^{(n)} + a(t)f(u) = 0, \qquad t \in (0, 1)$$
$$u(0) = 0, u'(0) = 0, \ldots, u^{(n-2)}(0) = 0, \qquad \alpha u(\eta) = u(1),$$

where $0 < \eta < 1, 0 < \alpha \eta^{n-1} < 1$. In particular, we establish the existence of at least one positive solution if f is either superlinear or sublinear by applying the fixed point theorem in cones due to Krasnoselkiĭ and Guo.
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1. Introduction

Investigation of positive solutions of nonlocal boundary value problems (BVPs), initiated by Il'in and Moiseev [1,2], has been recently addressed by various authors, for instance, [3–8]. Many authors refer to such problems as multipoint problems. Multipoint problems refer to a different family of boundary conditions in the study of disconjugacy theory [9]; hence, we choose to use the terminology nonlocal boundary conditions, introduced by Il'in and Moiseev, [1,2]. This work is motivated by Ma [7],

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* Corresponding author. Tel.: +1 937 229 2511; fax: +1 937 229 2566.
 E-mail addresses: eloe@notes.udayton.edu (P.W. Eloe), bashir_qau@yahoo.com (B. Ahmad).