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A NOTE ON RAPID CONVERGENCE OF APPROXIMATE SOLUTIONS FOR SECOND ORDER PERIODIC BOUNDARY VALUE PROBLEMS

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Abstract. In this paper, we develop a generalized quasilinearization technique for a nonlinear second order periodic boundary value problem and obtain a sequence of approximate solutions converging uniformly and quadratically to a solution of the problem. Then we improve the convergence of the sequence of approximate solutions by establishing the convergence of order k ($k \geq 2$).

1. Introduction

The technique of generalized quasilinearization developed by Lakshmikantham [1,2] has been found to be extremely useful to solve the nonlinear boundary value problems. A good number of examples can be seen in the text by Lakshmikantham and Vatsala [3] and in the references [4,5]. Recently, Mohapatra, Vajravelu and Yin [6] considered the periodic boundary value problem

$$-u''(x) = f(x, u(x)), \quad u(0) = u(\pi), \quad u'(0) = u'(\pi), \qquad x \in [0, \pi],$$

with the assumption that $\frac{\partial f}{\partial u} < 0$ and $\frac{\partial^2 f}{\partial u^2} \leq 0$ (condition (iii) of Theorem 3.3 [6]). In this paper, we replace the convexity (concavity) condition by a condition of the form $f \in C^2([0, \pi] \times R^2)$ and obtain a sequence of approximate solutions converging monotonically and quadratically to a solution of the problem. Then we discuss the convergence of order k ($k \geq 2$).

2. Preliminary results

We know that the homogeneous periodic boundary value problem

$$\begin{aligned} -u''(x) - \lambda u(x) = 0, \qquad & x \in [0, \pi], \\ u(0) = u(\pi), \qquad u'(0) = u'(\pi), & \end{aligned} \tag{2.1}$$

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