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Research Article

Existence of Solutions for Nonlinear Fractional Integro-Differential Equations with Three-Point Nonlocal Fractional Boundary Conditions

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We prove the existence and uniqueness of solutions for nonlinear integro-differential equations of fractional order $q \in (1,2]$ with three-point nonlocal fractional boundary conditions by applying some standard fixed point theorems.

1. Introduction

Fractional calculus (differentiation and integration of arbitrary order) is proved to be an important tool in the modelling of dynamical systems associated with phenomena such as fractal and chaos. In fact, this branch of calculus has found its applications in various disciplines of science and engineering such as mechanics, electricity, chemistry, biology, economics, control theory, signal and image processing, polymer rheology, regular variation in thermodynamics, biophysics, blood flow phenomena, aerodynamics, electro-dynamics of complex medium, viscoelasticity and damping, control theory, wave propagation, percolation, identification, and fitting of experimental data [1–4].

Recently, differential equations of fractional order have been addressed by several researchers with the sphere of study ranging from the theoretical aspects of existence and uniqueness of solutions to the analytic and numerical methods for finding solutions. For some recent work on fractional differential equations, see [5–11] and the references therein.

In this paper, we study the following nonlinear fractional integro-differential equations with three-point nonlocal fractional boundary conditions

$$\begin{aligned} D^q x(t) + f\left(t, x(t), (\phi x)(t), (\psi x)(t)\right) = 0, \quad 0 < t < 1,\ 1 < q \le 2, \\ D^{(q-1)/2} x(0) = 0, \quad a D^{(q-1)/2} x(1) + x(\eta) = 0, \quad 0 < \eta < 1, \end{aligned} \tag{1.1}$$