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Convergence theorems for strongly continuous semi-groups of asymptotically nonexpansive mappings

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Abstract

Let K be a nonempty closed convex subset of a real Banach space E. Let T {colon equals} {T (t) : t ∈ R+} be a strongly continuous semi-group of asymptotically nonexpansive mappings from K into K with a sequence {Lt} ⊂ [1, ∞). Suppose F (T) ≠ 0{combining long solidus overlay}. Then, for a given u_0 ∈ K and t_n > 0 there exists a sequence {u_n} ⊂ K such that u_n = (1 - a_n) T (t_n) u_n + a_n u_0, for n ∈ N such that {a_n} ⊂ (0, 1) and L_{t_n} - 1 < a_n, where t_n ∈ R+. Suppose, in addition, that E is reflexive strictly convex with a uniformly Gâteaux differentiable norm and that $\lim_{n \to \infty} t_n = \infty$, $\lim_{n \to \infty} a_n = \lim_{n \to \infty} \text{frac}(L_{t_n} - 1, a_n) = 0$. Then the sequence {$u_n$} converges strongly to a point of F (T). Moreover, it is proved that an explicit sequence {x_n} generated from x_1 ∈ K by $x_{n + 1}$ {colon equals} a_n u + (1 - a_n) T (t_n) x_n, n ≥ 1, converges to a fixed point of T. © 2009 Elsevier Ltd. All rights reserved.

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Asymptotically nonexpansive mappings; Fixed points; Nonexpansive mappings; Strongly continuous semi-groups of asymptotically nonexpansive mappings; Strongly continuous semi-groups of nonexpansive mappings