

**Document Type** : Thesis  
**Document Title** : COALGEBRAS AND COMODULES

الكوجبريات والكوموديوالات

**Document Language** : Arabic

**Abstract** : The subject under discussion, namely, "coalgebras and comodules" is one of the most recent developments in mathematics. In the past few decades this branch is extensively studied by numerous mathematicians, independently, and also jointly as an ingredient of a bialgebra or a Hopf algebra (also known as quantum groups). By the term coalgebra we mean coassociative coalgebra with a counit and comodules are counital. Initially, coalgebras are studied over fields and then shifted to commutative rings. Following [Mo:01] we worked in general on not necessarily commutative rings with identity. But most of the results we have proved are for commutative rings. Algebras and coalgebras are dual in nature in the sense that structurally they are dual and that their actions on abelian groups as modules and comodules are also dual. From a coalgebra  $C$  over some ring  $R$  we can get an algebra  $C^* = \text{Hom}_R(C, R)$  which is left dual to  $C$ . On the contrary, if  $A$  is an  $R$ -algebra, then its dual  $A^* = \text{Hom}_R(A, R)$  may not be an  $R$ -coalgebra. The obvious reason for this pathology is that in general for  $R$ -modules  $X$  and  $Y$ , is not isomorphic to. This isomorphism holds if  $R$  is a field and the  $R$ -vector spaces  $X$  and  $Y$  are finite dimensional or more generally  $A$  is finitely generated projective as an  $R$ -module. Finitely generated projective  $R$ -modules are termed as Cauchy modules. The condition of finiteness may be skipped by imposing other conditions on  $A$  so, if  $A$  is Cauchy as an  $R$ -module, then  $A^*$  can be given a structure of an  $R$ -coalgebra. There are many such dualities that one may encounter while studying these notions. In this thesis besides studying basic properties of coalgebras and comodules, cotensor products over coalgebras are also studied. Some results related to Morita theory are also included

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**Publishing Year** : 2007